中立时滞系统指数稳定的一种设计方法

郭新兰, 李文林

(河南师范大学 数学与信息科学学院, 河南 新乡 453007)

[摘要] 研究了一类时滞中立系统指数稳定问题, 设计一个基于观测器的状态反馈控制器保证误差系统指数稳定, 并且原系统也是指数稳定的. 通过解一个 LMI来解决设计增益问题, 克服了文献 [1]引入参数过多而又没有合适的选取参数方法的缺点. 最后, 以一个数值例子说明了设计方法的有效性和可行性.

[关键词] 中立系统, 指数稳定, Lyapunov函数, 反馈增益

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One Design Method of Exponential Stability for Neutral Delay System

GUO Xinlan, LI Wenlin

(School of Mathematics and Information Science, Henan Normal University, Henan Xinxiang 453007, China)

Abstract This paper studies the exponential stability problem for a class of neutral delay system, and designs a state feedback controller based on observer to guarantee the exponential stability for the error dynamic system, and the exponential stability for the original system. LMI is solved to settle the gain design question, to overcome the shortcomings in paper [1] that too many parameters are introduced but the proper ways to select these parameters are not given. Finally, an illustrative example is used to demonstrate the validity and feasibility of the proposed design procedure.

Key words neutral systems, exponential stability, Lyapunov functions, feedback gain

0 引言

近年来, 中立系统的稳定性分析和稳定化反馈问题引起了许多学者的兴趣, 产生了一系列相关的文章[1~5]. 在文献 [2]中, 研究了中立随机时滞系统的指数稳定; 在文献 [3] ~ [5]中, 线性及非线性随机系统的指数稳定也有相应的研究, 目前对于中立时滞系统指数稳定的设计仍然是一个重要和具有挑战性的问题; 在文献 [1]中, 利用奇异值分解的方法求反馈增益, 该方法引入了过多的参数 ϵ_i、δ_i, 怎样确定合适的参数值并没有给出具体的方法, 并且由于参数的引入需要利用不等式进行放大, 增加了保守性. 本文针对文献 [1]中提出的系统, 采用描述器的方法, 通过引入 Lyapunov函数, 把闭环系统指数稳定的充分条件归结为一个 LMI, 利用 LMI的可行解, 给出了反馈增益的表达式. 本文提出基于观测器的反馈增益的设计方法不需要引入标量参数, 且只用了两个不等式放大, 减少了所得结论的保守性. 最后, 以一个数值例子说明了本文所提出方法的有效性.

1 问题的描述与假设

考虑下列线性连续状态的时滞中立系统:

$$\dot{\boldsymbol{x}}(t) - \boldsymbol{J}\dot{\boldsymbol{x}}(t-h) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{A}_1\boldsymbol{x}(t-h) + \boldsymbol{B}\boldsymbol{u} \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \tag{2}$$

$$\boldsymbol{x}(t) = \varphi(t),\ t \in [-h, 0],\ \varphi := \{\varphi(s): -h \leqslant s \leqslant 0\} \in \boldsymbol{C}([-h, 0]; \boldsymbol{R}^n) \tag{3}$$

其中, $x(t) \in \boldsymbol{R}^n$ 是状态, $h > 0$是常时滞, $y(t) \in \boldsymbol{R}^p$ 是可测量输出, $u \in \boldsymbol{R}^m$ 为输入. $\boldsymbol{A}, \boldsymbol{A}_1, \boldsymbol{B}, \boldsymbol{J}$是已知适当维数常矩阵, $\boldsymbol{C}([-h, 0]; \boldsymbol{R}^n)$ 表示从 $[-h, 0]$ 到 $\boldsymbol{R}^n$ 的连续函数.

假设 1 矩阵 $\boldsymbol{J}$满足 $\boldsymbol{J} \neq \boldsymbol{0}$且 $\|\boldsymbol{J}\| < 1$.

设系统 (1) ~ (3) 的如下形式的观测器型动态输

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作者简介: 郭新兰 (1979-), 女, 硕士研究生, 主要从事自适应控制与鲁棒控制等方面的学习与研究. E-mail: xinlan-guo@163.com

通讯联系人: 李文林 (1949-), 教授, 博士, 主要从事滑模控制与自适应控制等方面的教学与研究. E-mail: hsdlwl@tom.com

出反馈控制:

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)-\boldsymbol{J}\dot{\hat{\boldsymbol{x}}}(t-h)=\boldsymbol{A}\hat{\boldsymbol{x}}(t)+\boldsymbol{A}_1\hat{\boldsymbol{x}}(t-h)+\\ \qquad \boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{L}[\boldsymbol{y}(t)-\boldsymbol{C}\hat{\boldsymbol{x}}(t)]\\ \boldsymbol{u}(t)=\boldsymbol{F}_1\hat{\boldsymbol{x}}(t)\end{cases} \tag{4}$$

其中, $\boldsymbol{F}_1$ 和 $\boldsymbol{L}$ 为待定的增益矩阵.

令观测误差为:

$$\boldsymbol{e}(t)=\boldsymbol{x}(t)-\hat{\boldsymbol{x}}(t) \tag{5}$$

从(1)和(4)可得:

$$\dot{\boldsymbol{e}}(t)-\boldsymbol{J}\dot{\boldsymbol{e}}(t-d)=(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})\boldsymbol{e}(t)+\boldsymbol{A}_1\boldsymbol{e}(t-h) \tag{6}$$

把(1)和(6)用描述器形式表示为:

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{y}(t)\\ \boldsymbol{y}(t)=\boldsymbol{J}\boldsymbol{y}(t-h)+(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{F}_1)\boldsymbol{x}(t)+\\ \qquad \boldsymbol{A}_1\boldsymbol{x}(t-h)-\boldsymbol{B}\boldsymbol{F}_1\boldsymbol{e}(t)\end{cases} \tag{7}$$

$$\begin{cases}\dot{\boldsymbol{e}}(t)=\boldsymbol{z}(t)\\ \boldsymbol{z}(t)=\boldsymbol{J}\boldsymbol{z}(t-h)+(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})\boldsymbol{e}(t)+\boldsymbol{A}_1\boldsymbol{e}(t-h)\end{cases} \tag{8}$$

为了导出本文的主要结果, 即下节的定理 1, 先给出如下引理.

引理 1[6] 如果 $\boldsymbol{f}\in\boldsymbol{R}^n$, $\boldsymbol{g}\in\boldsymbol{R}^n$, $\varepsilon>0$, 则有 $\boldsymbol{f}^{\mathrm{T}}\boldsymbol{g}+\boldsymbol{g}^{\mathrm{T}}\boldsymbol{f}\leqslant\varepsilon\boldsymbol{f}^{\mathrm{T}}\boldsymbol{f}+\varepsilon^{-1}\boldsymbol{g}^{\mathrm{T}}\boldsymbol{g}$.

引理 2[6] 对给定的对称矩阵 $\boldsymbol{S}=\begin{pmatrix}\boldsymbol{S}_{11} & \boldsymbol{S}_{12}\\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22}\end{pmatrix}$. 其中 $\boldsymbol{S}_{11}$ 是 $r\times r$ 维的, 以下 3个条件是等价的:

(i) $\boldsymbol{S}<\boldsymbol{0}$;

(ii) $\boldsymbol{S}_{11}<\boldsymbol{0}$, $\boldsymbol{S}_{22}-\boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}<\boldsymbol{0}$;

(iii) $\boldsymbol{S}_{22}<\boldsymbol{0}$, $\boldsymbol{S}_{11}-\boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}}<\boldsymbol{0}$

2 主要结果

定理 1 如果存在矩阵 $\boldsymbol{F}_1\in\boldsymbol{R}^{m\times n}$, $\boldsymbol{L}\in\boldsymbol{R}^{n\times p}$, $\boldsymbol{0}<\boldsymbol{R}_i\in\boldsymbol{R}^{n\times n}(i=1,2,3)$, $\beta>0$使得下述 LMI成立, 则由(7)~(8)组成的闭环系统指数稳定, 且稳定度不小于 β.

$$\begin{bmatrix}
\boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} & \boldsymbol{0} & \boldsymbol{M}_{15} & \boldsymbol{0} & \boldsymbol{M}_{17} & \boldsymbol{M}_{18} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & \boldsymbol{M}_{22} & \boldsymbol{M}_{23} & \boldsymbol{M}_{24} & -(\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}\boldsymbol{J} & \boldsymbol{M}_{26} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{29} & (\boldsymbol{L}\boldsymbol{C})^{\mathrm{T}} & (\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}} & \boldsymbol{0}\\
* & * & \boldsymbol{M}_{33} & \boldsymbol{0} & \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & \boldsymbol{M}_{44} & \boldsymbol{0} & \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & \boldsymbol{M}_{55} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & \boldsymbol{M}_{66} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}^{\mathrm{T}}\boldsymbol{R}_1\\
* & * & * & * & * & * & -\boldsymbol{I} & * & * & * & * & *\\
* & * & * & * & * & * & * & -\boldsymbol{I} & * & * & * & *\\
* & * & * & * & * & * & * & * & -\boldsymbol{I} & * & * & *\\
* & * & * & * & * & * & * & * & * & -\boldsymbol{I} & * & *\\
* & * & * & * & * & * & * & * & * & * & -\frac{1}{2}\boldsymbol{I} & *\\
* & * & * & * & * & * & * & * & * & * & * & -\boldsymbol{I}
\end{bmatrix}<\boldsymbol{0} \tag{9}$$

其中:

$\boldsymbol{M}_{11}=(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{F}_1)+(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}+2\beta\boldsymbol{I}+\boldsymbol{R}_3$,

$\boldsymbol{M}_{12}=-\boldsymbol{B}\boldsymbol{F}_1-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{F}_1$,

$\boldsymbol{M}_{13}=\boldsymbol{A}_1+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}_1+(\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}\boldsymbol{A}_1$,

$\boldsymbol{M}_{15}=\boldsymbol{J}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{J}+(\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}\boldsymbol{J}$,

$\boldsymbol{M}_{17}=(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}$,

$\boldsymbol{M}_{18}=(\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}$,

$\boldsymbol{M}_{22}=(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})+(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})^{\mathrm{T}}+2\beta\boldsymbol{I}+\boldsymbol{R}_2$,

$\boldsymbol{M}_{23}=-(\boldsymbol{B}\boldsymbol{F}_1)^{\mathrm{T}}\boldsymbol{A}_1$,

$\boldsymbol{M}_{24}=\boldsymbol{A}_1+(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})^{\mathrm{T}}\boldsymbol{A}_1$,

$\boldsymbol{M}_{26}=\boldsymbol{J}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{J}$,

$\boldsymbol{M}_{29}=(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})^{\mathrm{T}}$,

$\boldsymbol{M}_{33}=-\boldsymbol{R}_3e^{(-2\beta h)}+\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{A}_1$,

$\boldsymbol{M}_{44}=-\boldsymbol{R}_2e^{(-2\beta h)}+\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{A}_1$,

$\boldsymbol{M}_{55}=-e^{(-2\beta h)}\boldsymbol{I}+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J}$,

$\boldsymbol{M}_{66}=-\boldsymbol{R}_1e^{(-2\beta h)}+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{J}$.

证明 对(7)和(8)设计如下的 Lyapunov函数:

$$\boldsymbol{V}(t)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t)+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)+\boldsymbol{V}_1(t)+\boldsymbol{V}_2(t)+\boldsymbol{V}_3(t)+\boldsymbol{V}_4(t) \tag{10}$$

$$\boldsymbol{V}_1(t)=\int_h^t\boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{R}_1e^{2\beta(-t+s)}\boldsymbol{z}(s)\,\mathrm{d}s,$$

$$\boldsymbol{V}_2(t)=\int_h^t\boldsymbol{y}^{\mathrm{T}}(s)e^{2\beta(-t+s)}\boldsymbol{y}(s)\,\mathrm{d}s,$$

$$\boldsymbol{V}_3(t)=\int_h^t\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{R}_2e^{2\beta(-t+s)}\boldsymbol{e}(s)\,\mathrm{d}s,$$

$$\boldsymbol{V}_4(t)=\int_h^t\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_3e^{2\beta(-t+s)}\boldsymbol{x}(s)\,\mathrm{d}s$$

对(10)式求导得:

$$\dot{V}(t) = 2x^T(t)\dot{x}(t) + 2e^T(t)\dot{e}(t) + \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) \tag{11}$$

其中:

$$\dot{V}_1(t) = -2\beta\int_{t-h}^{t} z^T(s)R_1 e^{2\beta(-t+s)} z(s)\,ds + z^T(t)R_1 z(t) - z^T(t-h)R_1 e^{-2\beta h} z(t-h) \tag{12}$$

$$\dot{V}_2(t) = -2\beta\int_{t-h}^{t} y^T(s) e^{2\beta(-t+s)} y(s)\,ds + y^T(t)y(t) - y^T(t-h)e^{-2\beta h} y(t-h) \tag{13}$$

$$\dot{V}_3(t) = -2\beta\int_{t-h}^{t} e^T(s)R_2 e^{2\beta(-t+s)} e(s)\,ds + e^T(t)R_2 e(t) - e^T(t-h)R_2 e^{-2\beta h} e(t-h) \tag{14}$$

$$\dot{V}_4(t) = -2\beta\int_{t-h}^{t} x^T(s)R_3 e^{2\beta(-t+s)} x(s)\,ds + x^T(t)R_3 x(t) - x^T(t-h)R_3 e^{-2\beta h} x(t-h) \tag{15}$$

把(7)~(8)和(12)~(15)代入(11)可得:

$$\begin{aligned}\dot{V}(t) = {} & 2x^T(t)Jy(t-h) + 2x^T(t)(A + BF_1)x(t) + 2x^T(t)A_1x(t-h) - \\ & 2\beta e^T(t)e(t) - 2x^T(t)BF_1e(t) + 2e^T(t)Jz(t-h) + 2e^T(t)(A-LC)e(t) + 2e^T(t)A_1e(t-h) - \\ & 2\beta\int_{t-h}^{t} z^T(s)R_1e^{2\beta(-t+s)}z(s)\,ds + z^T(t)R_1z(t) - z^T(t-h)R_1e^{-2\beta h}z(t-h) - \\ & 2\beta\int_{t-h}^{t} y^T(s)e^{2\beta(-t+s)}y(s)\,ds + y^T(t)y(t) - \\ & 2\beta\int_{t-h}^{t} e^T(s)R_2e^{2\beta(-t+s)}e(s)\,ds + x^T(t)R_3x(t) - \\ & y^T(t-h)e^{-2\beta h}y(t-h) + e^T(t)R_2e(t) - \\ & 2\beta\int_{t-h}^{t} x^T(s)R_3e^{2\beta(-t+s)}x(s)\,ds - 2\beta x^T(t)x(t) - \\ & e^T(t-h)R_2e^{-2\beta h}e(t-h) - x^T(t-h)R_3e^{-2\beta h}x(t-h) + \\ & 2\beta x^T(t)x(t) + 2\beta e^T(t)e(t)\end{aligned} \tag{16}$$

其中:

$$\begin{aligned} z^T(t)R_1z(t) = {} & [z^T(t-h)J^T + e^T(t)(A-LC)^T + e^T(t-h)A_1^T]R_1[Jz(t-h) + (A-LC)e(t) + A_1e(t-h)] = \\ & z^T(t-h)J^TR_1Jz(t-h) + z^T(t-h)J^TR_1(A-LC)e(t) + z^T(t-h)J^TR_1A_1e(t-h) + \\ & e^T(t)(A-LC)^TR_1Jz(t-h) + e^T(t)(A-LC)^T(A-LC)e(t) + e^T(t)(A-LC)^TA_1e(t-h) + \\ & e^T(t-h)A_1^TR_1Jz(t-h) + e^T(t-h)A_1^TR_1(A-LC)e(t) + e^T(t-h)A_1^TR_1A_1e(t-h)\end{aligned} \tag{17}$$

$$\begin{aligned} y^T(t)y(t) = {} & [y^T(t-h)J^T + x^T(t)(A+BF_1)^T + x^T(t-h)A_1^T - e^T(BF_1)^T][Jy(t-h) + (A + BF_1)x(t) + A_1x(t-h) - BF_1e(t)] = \\ & y^T(t-h)J^TR_2Jy(t-h) + y^T(t-h)J^T(A + BF_1)x(t) + y^T(t-h)J^TA_1x(t-h) - \\ & y^TJ^TBF_1e(t) + x^T(t)(A + BF_1)^TJy(t-h) + x^T(t)(A + BF_1)^T(A + BF_1)x(t) + x^T(A + BF_1)^TA_1x(t-h) - \\ & x^T(t)(A + BF_1)BF_1e(t) + x^T(t-h)A_1^TJy(t-h) + x^T(t-h)A_1^T(A + BF_1)x(t) + x^T(t-h)A_1^TA_1x(t-h) - \\ & x^T(t-h)A_1^TBF_1e(t) - e^T(t)(BF_1)^TJy(t-h) - e^T(t)(BF_1)^T(A + BF_1)x(t) - \\ & e^T(t)(BF_1)^TA_1x(t-h) + e^T(t)(BF_1)^T(BF_1)e(t)\end{aligned} \tag{18}$$

利用引理1,有:

$$-2x^T(t)(BF_1)^T(BF_1)e(t) \leqslant x^T(t)(BF_1)^T(BF_1)x(t) + e^T(t)(BF_1)^T(BF_1)e(t) \tag{19}$$

$$-2e^T(t)(LC)^TR_1Jz(t-h) \leqslant e^T(LC)^T(LC)e(t) + z^T(t-h)J^TR_1R_1Jz(t-h) \tag{20}$$

记: $\zeta^T = [x^T(t),\ e^T(t),\ x^T(t-h),\ e^T(t-h)\ y^T(t-h),\ z^T(t-h)]$

把(17)~(20)代入(16)式可得:

$$\dot{V}(t) \leqslant \zeta^T N\zeta - 2\beta V(t) \tag{21}$$

记 $N = (N_{ij}),\ i, j = 1, \cdots, 6$

$N_{11} = A + BF_1 + (A + BF_1)^T + 2\beta I + (A + BF_1)^T(A + BF_1) + R_3 + (BF_1)^T(BF_1)$,

$N_{12} = -BF_1 - A^TBF_1$,

$N_{13} = A_1 + A^TA_1 + (BF_1)^TA_1$,

$N_{14} = 0$,

$N_{15} = J + A^TJ + (BF_1)^TJ$,

$N_{16} = 0$,

$N_{22} = A - LC + (A - LC)^T + 2\beta I + R_2 + (A - LC)^T(A - LC) + (LC)^T(LC) + 2(BF_1)^T(BF_1)$,

$N_{23} = -(BF_1)^TA_1$, $N_{24} = A_1 + (A - LC)^TA_1$,

$N_{25} = -(BF_1)^TJ$, $N_{26} = J + A^TR_1J$,

$N_{33} = -R_3e^{-2\beta h} + A_1^TA_1$,

$N_{34} = 0$, $N_{35} = A_1^TJ$, $N_{36} = 0$,

$N_{44} = -R_2e^{-2\beta h} + A_1^TR_1A_1$,

$N_{45} = 0$, $N_{46} = A_1^TR_1J$,

$N_{55} = -e^{-2\beta h}I + J^TJ$, $N_{56} = 0$,

$N_{66} = -R_1e^{-2\beta h} + J^TR_1J + J^TR_1R_1J$.

即:

$$N = \begin{bmatrix} N_{11} & N_{12} & N_{13} & 0 & N_{15} & 0 \\ * & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} \\ * & * & N_{33} & 0 & N_{35} & 0 \\ * & * & * & N_{44} & 0 & N_{46} \\ * & * & * & * & N_{55} & 0 \\ * & * & * & * & * & N_{66} \end{bmatrix} \quad (22)$$

如果存在矩阵 $L \in R^{n\times p}$, $F_1 \in R^{m\times n}$, $0 < R_i \in R^{n\times n}$, $(i = 1, 2, 3)$, $\beta > 0$, $h = 0.1778$, 使得 $N < 0$成立, 则有: $\dot{V}(t) \leqslant -2\beta V(t)$, 利用引理 2, $N < 0$等价于 (9) 式成立, 即当 (9) 式成立时, 有 $\dot{V}(t) < -2\beta V(t) < 0$ 参考 [6] 以及假设 (1) 知由 (7) ~ (8) 组成的闭环系统是指数稳定的, 并且稳定度 β 可以通过解 LMI(9) 得到. 证毕.

3 数值例子

考虑系统 (1) - (2):

$$A = \begin{pmatrix} -1.25 & 0 \\ 0 & -1.10 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} 0.01 & 0.1 \\ -0.023 & 0.1 \end{pmatrix}, C = (1 \quad 0), B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$J = 0.0001 I_2.$$

通过解 LMI(9) 可得:

$$R_1 = \begin{pmatrix} 0.6432 & -0.0002 \\ -0.0002 & 0.6432 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 0.4090 & -0.0003 \\ -0.0003 & 0.4344 \end{pmatrix},$$

$$R_3 = \begin{pmatrix} 0.4193 & -0.0040 \\ -0.0040 & 0.4347 \end{pmatrix},$$

$$L = \begin{pmatrix} -0.2984 \\ 0.0002 \end{pmatrix},$$

$$F_1 = \begin{pmatrix} 0.0011 \\ 0.0295 \end{pmatrix}, \beta = 0.0712.$$

L和F_1即为所求的反馈增益, β为所求的系统稳定度.

4 结论

本文针对连续时滞中立系统讨论了其基于观测器的状态反馈控制, 该控制能够保证原系统以及观测误差都是指数稳定的, 本文提出的方法是用描述器来实现的, 避免引入过多的参数, 克服了文献 [1]引入参数过多的缺点, 降低了结果的保守性, 并且反馈增益的存在性及指数稳定度都可以通过一个 LMI解出, 简洁易行. 最后, 以一个数值例子验证了本文所提出方法的有效性和可行性.

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